Finding Eigenvalues and Eigenvectors

# Introduction

[Link](https://nptel.ac.in/courses/111107062/module3/lecture1/lecture1.pdf) 1 and [link](https://nptel.ac.in/courses/111107062/module3/lecture2/lecture2.pdf) 2 give good tutorials on how to get the maximum and minimum eigenvalues and eigenvectors by iterative processes. However, I have noticed that eigenvalue converges to its final value much more quickly than the eigenvector. In this short document I will demonstrate how to get eigenvectors that are in agreement with their eigenvalues.

# Math

It is obvious that eigenvectors can be multiplied by any constant and they still are perfectly good solutions of the eigenvector equation. There I choose to make the first element of any eigenvector unity (1) and scale the remainder of the elements with respect to this first element. To demonstrate the procedure for getting the remaining elements, I will assume a 3x3 matrix but it will work for any square matrix:



We want to solve equation for *v*2 and *v3*. To do that we form the matrix product for the lower two rows times our eigenvector and subtract out  from the resulting diagonal elements.



We can solve for v2 and v3 by inverting the 2x2 matrix as shown below:





Since the matrix product on the left side of equation is the unit matrix, we are left with the equation:



which results in an eigenvector that agrees perfectly with our Power Method eigenvalue.

Again I want to emphasize that this method for eigenvector calculation works for any size square matrix.