Hohmann Transfers

# Introduction

This will be a paraphrase of the main reference [1](https://ocw.mit.edu/courses/aeronautics-and-astronautics/16-07-dynamics-fall-2009/lecture-notes/MIT16_07F09_Lec17.pdf) that I found a little hard to follow. Figure 1 shows the pertinent variables that will be used.

r11

r221

r1+r2

*Figure 1: Circular orbits r1 and r2 as well as the elliptical Hohmann transfer orbit*.

Obviously the semi-major axis of the ellipse is  . This means that the energy needed for the elliptical transfer orbit is more than that of the inner orbit and smaller than that of the outer orbit. The velocity of any circular orbit can be obtained from the requirement that the centripetal force equal the gravitational attraction





Where  is the angular rate of the mass m in its orbit, *m* is its mass, *r* is the radius of the circular orbit, and *GM* is the gravitational constant, *G*, times the mass, *M*, of the planet at the center of the orbit. For a circular orbit the speed, *v*, of mass m is . We can now evaluate the kinetic energy, E, of the circular orbit using equation



For convenience we will now use specific kinetic energy which is energy per unit mass.

Using equation we can state the specific energies, e, of the inner and outer circular orbits as



The corresponding speeds of these 2 orbits are



For the transfer orbit, conservation of angular momentum requires that



Where *l* is the mass specific angular momentum.

At perigee the total specific energy is



And this energy must be the same at the apogee



We can multiply equation by  and equation by and obtain





Dividing both equations by their radii squared we obtain the following:





We may now set equations and equal obtaining



Now solve for *l2*



Now we use equation and equation to solve for *v*perigee and *vapogee* .

 

Equation is the solution for ***any*** elliptical orbit. For the Hohmann transfer orbit we have referring to Figure 1:



The speed change needed for perigee and apogee can be calculated using equations and equations



Here  is the speed change needed to go from the circular orbit of radius *r1* to the Hohmann transfer orbit and  is the speed change needed to go from the Hohmann transfer orbit to the circular orbit of radius *r2*. These speed changes need to be provided by very short burns when the spacecraft is very near the perigee and apogee, respectively, of the transfer orbit. Ideally the apogee burn should start a short time before reaching apogee and end at the same short time after apogee.

The time between the start of the transfer orbit and reaching apogee is the Kepler expression for ½ of an orbital period.



Now we can write eccentricity as



And semi-minor axis, b, as



Where



The rate of angular rotation with respect to M is given by



Where



Finally we can write equation as



Therefore we can compute the time variation of ** and *r* by numerically integrating equation .

Having obtained ** and *r* for any time t, we can use the following equations to compute the Cartesian coordinates (x,y).

